

Mark Scheme (Results)

June 2015

Pearson Edexcel International A Level in Statistics 2 (WST02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

June 2015 WMST02/01 Statistics 2 Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\{P(X > 4) = \}1 - F(4)$ 1- F(4) seen or used	M1
	$\left\{ = 1 - \frac{3}{5} \right\} = \frac{2}{5}$ $\frac{2}{5} \text{ or } 0.4$ $P(3 < X < a) = 0.642$	A1 [2]
(b)	F(3 < X < a) = 0.042 F(a) - F(3) = 0.642 F(a) - F(3) = 0.642	M1 o.e.
	$F(a) - \frac{1}{20}(3^2 - 4) = 0.642 $ { $\Rightarrow F(a) = 0.892$ } Correct equation	A1 o.e.
	$\frac{1}{5}(2a-5) - \frac{1}{20}(3^2-4) = 0.642 \Rightarrow a = \dots$ Solving this equation o.e., leading to $a = \dots$ (or $x = \dots$). Follow through their F(3)	dM1
	$\left\{ \frac{1}{5}(2a-5) = 0.892 \implies \right\} \ a = 4.73 \qquad a = 4.73 \text{ (or } x = 4.73 \text{)}$	A1 cao
(b)	Alternative Method for Part (b)	[4]
	$\int_{3}^{4} \left(\frac{1}{10}x\right) \{dx\}$ Correct expression for finding the probability between $x = 3$ and $x = 4$	M1
	$\left\{ = \left[\frac{x^2}{20} \right]_3^4 \right\} = \frac{4^2}{20} - \frac{3^2}{20} \left\{ = \frac{7}{20} \right\}$ Correct $\frac{4^2}{20} - \frac{3^2}{20}$, simplified or un-simplified.	A1
	$\int_{3}^{4} \left(\frac{1}{10}x\right) \left\{ dx \right\} + \int_{4}^{a} \left(\frac{2}{5}\right) \left\{ dx \right\} = 0.642 \Rightarrow a = \dots$ Writes a correct equation and attempts to solve leading to $a = \dots$ (or $x = \dots$)	dM1
	$\left\{ \frac{7}{20} + \frac{2}{5}a - \frac{8}{5} = 0.642 \Rightarrow \right\} a = 4.73 \qquad a = 4.73 \text{ (or } x = 4.73 \text{)}$	A1 cao
(c)		[4]
(c)	$f(x) = \frac{d}{dx} \left(\frac{1}{20} (x^2 - 4) \right) = \frac{1}{10} x$ Attempt at differentiation. See notes.	M1
	At least one of $-x$ or $-$	A1
	$f(x) = \frac{1}{dx} \left(\frac{1}{5} (2x - 5) \right) = \frac{1}{5}$ Both $\frac{1}{10} x$ and $\frac{2}{5}$	A1
	$f(x) = \begin{cases} \frac{1}{10}x, & 2 \le x \le 4 \\ \frac{2}{5}, & 4 < x \le 5 \end{cases}$ This mark is dependent on M1 All three lines with limits correctly followed through from their F'(x) $0, \text{ otherwise}$	dB1ft
		[4] 10

		Question 1 Notes
1. (a)	M1	1-F(4) seen or used.
	Note	Can be implied by either $1 - \frac{3}{5}$ or $1 - \frac{1}{5}(2(4) - 5)$ or $1 - \frac{1}{20}(4^2 - 4)$
		The probability statements $1 - P(X \le 4)$ or $1 - P(X \le 4)$ are not sufficient for M1
	A1	$\frac{2}{5}$ or 0.4
<i>a</i> >	Note	Give M1A1 for the correct answer from no working.
(b)	NOTE	In part (b), candidates are allowed to write • $F(a)$ as either $P(X < a)$ or $P(X \le a)$. Also condone $F(a)$ written as $F(x)$
		• $F(3)$ as either $P(X < 3)$ or $P(X \le 3)$
	M1	For writing $F(a) - F(3) = 0.642$ or equivalent (see NOTE above)
	A1	For an un-simplified $F(a) - \frac{1}{20}(3^2 - 4) = 0.642$ or equivalent (see NOTE above)
	Note	Give 1 st M1 1 st A1 for $F(a) = 0.892$ or $P(X \ge a) = 0.108$
	SC	Allow SC 1 st M1 1 st A1 for $\frac{1}{20}(a^2-4) - \frac{1}{20}(3^2-4) = 0.642$
	Note	Give 1 st M0 for $F(a-1) - F(3) = 0.642$ o.e. without a correct acceptable statement
	dM1	dependent on the FIRST method mark being awarded.
		Attempts to solve $\frac{1}{5}(2a-5)$ – "their F(3)" = 0.642 leading to $a =$ (or $x =$)
	Note	dM1 can be given for either $\frac{1}{5}(2a-5) = 0.892$ or $1 - \frac{1}{5}(2a-5) = 0.108$ leading to $a =$ (or $x =$)
	A1	a = 4.73 (or $x = 4.73$) cao
	Note	Give M0A0M0A0 for $F(a) - (1 - F(3)) = 0.642 $ { $\Rightarrow F(a) = 1.392$ }
	Note	
	Note	Give M0A0M0A0 for $\int_{3}^{a} \left(\frac{1}{10}x\right) dx = 0.642$ (this solves to give awrt 4.67)
(c)	M1	At least one of either
		$\frac{1}{20}(x^2 - 4) \rightarrow \pm \alpha x \pm \beta, \ \alpha \neq 0, \beta \text{ can be } 0$
		$\frac{1}{20}(x^2 - 4) \to \pm \alpha x \pm \beta, \ \alpha \neq 0, \beta \text{ can be } 0$ $\frac{1}{5}(2x - 5) \to \pm \delta, \ \delta \neq 0$
	1 st A1	At least one of $\frac{1}{10}x$ or $\frac{2}{5}$. Can be simplified or un-simplified.
	2 nd A1	Both $\frac{1}{10}x$ and $\frac{2}{5}$. Can be simplified or un-simplified.
	dB1ft	dependent on the FIRST method mark being awarded. All three lines with limits correctly followed through from their $F'(x)$
	Note	Condone the use of $<$ rather than \le or vice versa.
	Note	0, otherwise is equivalent to 0, $x < 2$ and 0, $x > 5$
	Note	In part (c), accept f being expressed consistently in another variable eg. u

Question Number	Scheme			
2. (a)	$X \sim Po(8)$			
	$\left\{ P(X \neq 8) \right\} = 1 - P(X = 8)$ 1 - P(X = 8), can be implied	M1		
	= 0.860413 or 0.8605	A1		
(b)	$X \sim \text{Po}(8)$	[2]		
· /	${P(X \ge 8)} = 1 - 0.453$ $1 - 0.453$ or awrt 0.547	B1		
	$\left\{ \left[P(X \ge 8) \right]^4 \right\} = (1 - 0.453)^4 \left\{ = (0.547)^4 \right\}$ Applying $\left[\text{their } P(X \ge 8) \right]^4$	M1		
	= 0.089526 0.09 or awrt 0.090	A1		
(-)	W	[3]		
(c)	Y = number of chocolate chips in the 9 biscuits Normal or N	M1		
	${Y \sim Po(72) \approx }Y \sim N(72, 72)$	A1		
	$\{P(Y > 75)\} \approx P(Y > 75.5)$ For either 74.5 or 75.5	M1		
	(75.5 72) Standardising (\pm) with their mean,			
	$= P\left(Z > \frac{75.5 - 72}{\sqrt{72}}\right)$ Standardising (±) with their mean, their standard deviation and either 75.5 or 75 or 74.5	M1		
	= P(Z > 0.41) = 1 - 0.6591			
	= 0.3409 (from calculator 0.339994) awrt 0.341 or awrt 0.340	A1		
(d)	$H_0: \lambda = 1.5, H_1: \lambda > 1.5 \text{ or } H_0: \lambda = 6, H_1: \lambda > 6$ Both hypotheses are stated correctly	[5] B1		
	{Under H_0 , for 4 hours} $X \sim Po(6)$			
	Probability Method $P(X \ge 11) = 1 - P(X \le 10)$ Critical Region Method $P(X \le 9) = 0.9161$ or $P(X \ge 10) = 0.0839$			
	$= 1 - 0.9574 P(X \le 10) = 0.9574 or P(X \ge 11) = 0.0426$	M1		
	Note: Award 1 st M1 for the use of $X \sim Po(6)$			
	$P(X \ge 11) = 0.0426$ $CR : X \ge 11$ $Either P(X \ge 11) = 0.0426 \text{ or } CR : X \ge 11 \text{ or } CR : X > 10$	A1		
	Reject H ₀ or significant or 11 lies in the CR dependent on previous M See notes	dM1		
	Conclude either			
	 The <u>rate of sales</u> of packets of biscuits has <u>increased</u>. The <u>mean</u> number of packets of biscuits <u>sold</u> has in context. 	A1 cso		
	increased.	[5]		
		15		

		Question 2 Notes
2. (a)	M1	$1 - P(X = 8)$ or $P(X < 8) + P(X > 8)$ or $P(X \le 7) + P(X \ge 9)$
	Note	Can be implied by either $1 - \frac{e^{-8}8^8}{8!}$ or $1 - \left(P(X \le 8) - P(X \le 7)\right)$
		or $1 - (0.5925 - 0.4530)$ or $1 - 0.1395$ or $P(X \le 7) + 1 - P(X \le 8)$
	A1	0.86 or awrt 0.860 or awrt 0.861
(b)	B1	1-0.453 or awrt 0.547 (Note: calculator gives 0.5470391905)
	M1	Applying [their $P(X \ge 8)$] ⁴
	A1	0.09 or awrt 0.090 (Note: calculator gives 0.08955168526)
(c)	1 st M1	For writing N or for using a normal approximation.
	1st A1	For a correct mean of 72 and a correct variance of 72
	Note	1 st M1 and/or 1 st A1 may be implied in applying the standardisation formula
	2 nd M1	For either 74.5 or 75.5 (i.e. an attempt at a continuity correction)
	3 rd M1	Standardising (\pm) with their mean, their standard deviation and either 75.5 or 75 or 74.5
	Note	Award 2 nd M1 3 rd M0 for $\frac{75.5-72}{72}$ from a correct $Y \sim N(72, 72)$
	Note	You can recover the 1 st A1 in part (c) for $N(72, \sqrt{72}) \Rightarrow z = \frac{75.5 - 72}{\sqrt{72}}$
	2 nd A1	awrt 0.341 or awrt 0.340. (Note: calculator gives 0.339994)
(d)	B1	$H_0: \lambda = 1.5, H_1: \lambda > 1.5$ correctly labelled or $H_0: \lambda = 6, H_1: \lambda > 6$.
	Note	Allow μ used instead of λ
	Note	B0 for either $H_0 = 6$, $H_1 > 6$ or $H_0 : x = 6$, $H_1 : x > 6$ or $H_0 : p = 6$, $H_1 : p > 6$
	1 st M1	For use of $X \sim Po(6)$ (may be implied by 0.9161, 0.9574, 0.9799, 0.0839, 0.0426 or
		0.0201). Condone by $\frac{e^{-6}(6)^{11}}{11!}$. Allow any value off the Po(6) tables.
	1st A1	For either $P(X \ge 11) = 0.0426$ or $CR: X \ge 11$ or $CR: X > 10$ Condone $CR \ge 11$
	Note	Award 1 st M1 1 st A1 for writing down CR: $X \ge 11$ or CR: $X > 10$ from no working.
	Note	Give A0 stating CR : $P(X \ge 11)$
	2 nd dM1	dependent on the FIRST method mark being awarded.
		For a correct follow through comparison based on their probability or CR and their
		significance level compatible with their <i>stated</i> alternative hypothesis.
		Do not allow non-contextual conflicting statements. Eg. "significant" and "accept H_0 ".
	Note	M1 can be implied by a correct contextual statement.
	Note	Give final M0A0 for $P(X = 11) = 0.9799 - 0.9574 = 0.0225 \Rightarrow \text{Reject H}_0$, etc.
	Note	Give final M0A0 for $P(X \le 11) = 0.9799 \Rightarrow Accept H_0$, etc
	2 nd A1	Award for a correct solution only with all previous marks in part (d) being scored.
		Correct conclusion which is in context, using either the words
		rate of sales and increased or mean sold and increased

Question Number	Scheme	Marks
3. (a)	$\{f(x)\}$ A horizontal line drawn above the x -axis in the first quadrant	B1
	$\frac{1}{c}$ C	dB1
(b)	$E(X) = \frac{3c}{2}$ $E(X) = \frac{3c}{2}$, simplified or un-simplified.	[2] B1
	$\left\{ E(X^2) = \right\} \int_{c}^{2c} \left(\frac{1}{2c - c} x^2 \right) \left\{ dx \right\} $ $= \begin{cases} \sum_{c}^{2c} \left(\frac{1}{2c - c} x^2 \right) \left\{ dx \right\} \end{cases}$ equivalent to $\frac{1}{c}$. (Limits are required)	M1
	$= \left[\frac{1}{c} \left(\frac{x^3}{3}\right)\right]_{\{c\}}^{\{2c\}} \qquad \pm Ag(c)x^2 \to \pm Bg(c)x^3, \ A \neq 0, \ B \neq 0$ (Ignore limits for this mark)	M1
	$= \left(\frac{(2c)^3}{3c} - \frac{c^3}{3c}\right) \left\{ = \frac{7c^2}{3} \right\}$ dependent on first M mark. Applies limits of 2c and c to an <i>integrated</i> function in x and subtracts the correct way round.	dM1
	$Var(X) = E(X^{2}) - (E(X))^{2}$ $= \frac{7c^{2}}{3} - \left(\frac{3c}{2}\right)^{2}$ dependent on first M mark. Applying the variance formula correctly with their E(X)	dM1
	$= \frac{c^2}{12} * $ Correct proof	A1
(c)	Correct un-simplified (or simplified) inequality statement. Can be implied by $X > \frac{4c}{3}$	[6] M1
	$\Rightarrow X > 4c - 2X \Rightarrow 3X > 4c$	
	$\Rightarrow X > \frac{4c}{3}$ Rearranges $X > 2(2c - X)$ to give $X >$ or $X <$ See notes	dM1
	$\left\{ P(X > 2(2c - X)) = P\left(X > \frac{4c}{3}\right) \right\} = \frac{2}{3}$	A1
		[3] 11
	Note: In (c), give M2 for either $X > \frac{4c}{3}$ or $P\left(X > \frac{4c}{3}\right)$ or $1 - P\left(X < \frac{4c}{3}\right)$	

		Question 3 Notes
3. (a)	1st B1	A horizontal line drawn above the x-axis in the first quadrant
	2 nd dB1	dependent on the FIRST B mark being awarded.
		Labels of c , $2c$ and $\frac{1}{c}$, marked on the graph.
	Note	Allow the label $\frac{1}{2c-c}$ as an alternative to $\frac{1}{c}$
	Note	Ignore $\{O\}$, $\{x\}$ and $\{f(x)\}$
(b)	B1	$E(X) = \frac{3c}{2}$, simplified or un-simplified. This mark can be implied.
	Note	B1 can be given for an un-simplified $\left(\frac{(2c)^2}{c}\right) - \left(\frac{c^2}{c}\right)$ or $\frac{3c^2}{2c}$ or $2c - \frac{c}{2}$ etc.
	Note	$\int_{c}^{2c} \frac{1}{c} x dx \text{ or } \left[\frac{x^2}{2c}\right]_{c}^{2c} \text{ are not sufficient for B1.}$
	1 st M1	Correct E(X^2) expression of $\int_{c}^{2c} x^2 f(x) \{dx\}$ where $f(x)$ is equivalent to $\frac{1}{c}$.
	Note	Must have limits of $2c$ and c . Note the dx is not required for this mark.
	2 nd M1	$\pm Ag(c)x^2 \rightarrow \pm Bg(c)x^3$, $A \neq 0$, $B \neq 0$, where $g(c)$ is a function of c
	Note	Limits are not required for the second 2 nd M1 mark.
	3 rd dM1	dependent on the FIRST method mark being awarded. Applies limits of $2c$ and c to an integrated function in x and subtracts the correct way round.
	4 th M1	dependent on the FIRST method mark being awarded. Applying the variance formula correctly with their follow through $\mathrm{E}(X)$.
	Note	Allow 4 th M1 for $\left\{ \operatorname{Var}(X) = \right\} \int_{c}^{2c} \left(\frac{1}{2c - c} x^2 \right) \left\{ dx \right\} - \left(\int_{c}^{2c} \left(\frac{1}{2c - c} x \right) \left\{ dx \right\} \right)^2$
	A1	Correctly proves that $Var(X) = \frac{c^2}{12}$. Note: Answer is given
(c)	1 st M1	For writing down a correctly un-simplified (or simplified) inequality statement. Eg: $X > 2(2c - X)$ or $P(X > 2(2c - X))$ (Note: "P" is not required for this mark)
	2 nd dM1	dependent on the FIRST method mark being awarded. Rearranges to give $P(X > \pm \alpha c)$ or $P(X < \pm \alpha c)$ or $X > \pm \alpha c$ or $X < \pm \alpha c$, $\alpha \neq 0$
	Note Note	"P" is not required for these cases above Also allow, with P, the statements $1 - P(X < \pm \alpha c)$ or $1 - P(X > \pm \alpha c)$, $\alpha \neq 0$
	NOTE	Give M2 for either $X > \frac{4c}{3}$ or $P\left(X > \frac{4c}{3}\right)$ or $1 - P\left(X < \frac{4c}{3}\right)$
	A1	$\frac{2}{3}$ or $\frac{4}{6}$ or $0.\dot{6}$
	Note	Give M1M1A1 for a final answer of $\frac{2}{3}$ from any working.

Question Number	Scheme	Marks			
3.	Alternative Method 1 for Part (b)				
(b)	$\left\{ \operatorname{Var}(X) = \right\}$				
	Implied $E(X) = \frac{3c}{2}$	B1			
	$\int_{c}^{2c} x^{2} f(x) \{dx\} \text{ where } f(x) \text{ is equivalent to } \frac{1}{c}.$ $\int_{c}^{2c} \left(\frac{1}{2c-c} \left(x-\frac{3}{2}c\right)^{2}\right) \{dx\} \qquad (Limits \text{ are required}).$	1 st M1			
	Applies $\int_{c}^{2c} \left(\frac{x - \frac{1}{2}c}{2c - c} \right)^{2c} \left\{ dx \right\} $ where $f(x)$ is a is equivalent to $\frac{1}{c}$. (Limits are required)	4 th dM1			
	$= \frac{1}{c} \left[\frac{1}{3} \left(x - \frac{3c}{2} \right)^3 \right]_{\{c\}}^{\{2c\}} $ $ \pm Ag(c)(x - \delta)^2 \to \pm Bg(c)(x - \delta)^3, $ $A, B, \delta \neq 0 \text{ (Ignore limits for this mark)} $	2 nd M1			
	$= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$ $= \frac{1}{3c} \left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 - \left$	3 rd dM1			
	$= \frac{1}{3c} \left(\frac{c^3}{4} \right) = \frac{c^2}{12} *$ Correct proof	A1 [6]			
	Alternative Method 2 for Part (b)				
(b)	$\left\{ Var(X) = \right\}$				
	$\int_{c}^{2c} \left(\frac{1}{2c - c} \left(x - \frac{3}{2}c \right)^{2} \right) \left\{ dx \right\}$ Award as in Alt. Method 1	B1 1 st M1 4 th M1			
	$= \frac{1}{c} \int_{c}^{2c} \left(x^2 - 3cx + \frac{9}{4}c^2 \right) \left\{ dx \right\}$				
	$= \frac{1}{c} \left[\frac{1}{3} x^3 - \frac{3}{2} c x^2 + \frac{9}{4} c^2 x \right]_{\{c\}}^{\{2c\}} $ $\pm Ag(c)(x - \delta)^2 \rightarrow \pm Bg(c)(\pm \alpha x^3 \pm \beta x^2 \pm \delta x)^3,$ $A, B, \alpha, \beta, \delta \neq 0 \text{ (Ignore limits for this mark)}$	2 nd M1			
	$= \frac{1}{c} \left(\left(\frac{1}{3} (2c)^3 - \frac{3}{2} c (2c)^2 + \frac{9}{4} c^2 (2c) \right) - \left(\frac{1}{3} (c)^3 - \frac{3}{2} c (c)^2 + \frac{9}{4} c^2 (c) \right) \right)$ As earlier				
	$= \frac{1}{c} \left(\left(\frac{8}{3}c^3 - 6c^3 + \frac{9}{2}c^3 \right) - \left(\frac{1}{3}c^3 - \frac{3}{2}c^3 + \frac{9}{4}c^3 \right) \right)$				
	$=\frac{1}{c}\left(\left(\frac{7}{6}c^3\right) - \left(\frac{13}{12}c^3\right)\right) = \frac{1}{c}\left(\frac{c^3}{12}\right)$				
	$= \frac{c^2}{12} * $ Correct proof	A1			
		[6]			

Question Number		Scheme		Marks
4. (a)	$\begin{cases} P(X=0 \mid k=5) = 0.0067 & \text{probabilities or awrt 3.7} \\ \left\{ e^{-k} < 0.025 \Rightarrow k > \right\} 3.688 \end{cases}$		B1	
	$P(X \leq$	$8 \mid k = 3$) = 0.9962, $P(X \ge 9 \mid k = 3) = 0.0038$ $8 \mid k = 4$) = 0.9786, $P(X \ge 9 \mid k = 4) = 0.0214$ $8 \mid k = 5$) = 0.9319, $P(X \ge 9 \mid k = 5) = 0.0681$	Both $P(X = 0) = 0.0183$ or awrt 3.7 and either $P(X \ge 9) = 0.0214$ or $P(X \le 8) = 0.9786$	B1
	Both ta	ils less than 2.5% when $\underline{k} = 4$	Final answer given as $\underline{k} = 4$	B1
(1.)	A atual	sig. level = $0.0214 + 0.0183$		[3]
(b)	Actual	= 0.0397		M1
		_ 0.0397	0.0397	A1 cao
				[2]
		Question 4 No	ntes	
4. (a)		Question 4 110	otes	
10 (a)	1st B1	For any of 0.0498, 0.0183, 0.0067, 0.9962, 0.9 or awrt 3.7 seen in their working.	9786, 0.9319, 0.0038, 0.0214	, 0.0681
	2 nd B1	For both $P(X = 0) = 0.0183$ or awrt 3.7 and ei	ither $P(X \ge 9) = 0.0214$ or $P($	$X \leqslant 8) = 0.9786$
	Note	These must be written as probability statemen	ts.	
	3 rd B1	Final answer given as $k = 4$. Also allow $\lambda =$	 - Δ	
	Note	Do not recover working for part (a) in part		
(b)	M1	For the addition of two probabilities for two ta		
	A1	0.0397 cao		

Question Number			S	cheme			Marks
5.	$Y = \frac{2X_1 + X_2}{3} $ w	here	P(X = x)	6 0.35	9 0.65		
	Note: Yo	ou can ma	rk parts (a) and (b) to	gether for	this question.	
(a)	$\frac{2(6)+6}{3} = 6$	<u>2(9)</u> 3	$\frac{+9}{} = 9$		At least thre	ee correct values for y of either 6, 7, 8 or 9	B1
	$\frac{2(6)+9}{3} = 7$	$\frac{2(9)}{3}$	$\frac{+6}{}$ = 8	Correc	t values for	y of 6, 7 8 and 9 only	В1
							[2]
(b)	$\begin{cases} (6,6) \Rightarrow P(Y = \\ (6,9) \Rightarrow P(Y =) \end{cases}$,		(0.6		one of either $(0.35)^2$, .35)(0.65) or $(0.65)^2$	M1
	$\begin{cases} (9,6) \Rightarrow P(Y = \\ (9,9) \Rightarrow P(Y = \\ \end{cases}$,		(0.6		two of either $(0.35)^2$, .35)(0.65) or $(0.65)^2$	M1
		′				_	
	sample	(6, 6)	(6, 9)	(9, 6)	(9, 9)	See notes	A1
	P(Y = y)	6 0.1225	7 0.2275	8 0.2275	9 0.4225	At least 3 correct	A1
	or $P(Y = y)$	$\frac{49}{400}$	$\frac{91}{400}$	$\frac{91}{400}$	$\frac{169}{400}$	See notes	B1ft
							[5]
(c)	$\left\{ \mathbf{E}(Y) \right\} = 6(0.12)$	25) + 7(0.	2275) + 8(0	.2275) + 9(0.4225) = 7	$7.95 \text{ or } \frac{159}{20}$	M1;A1 cao
							[2] 9
(c)	Alternative Met	hod for F	art (c)				
	$\begin{cases} E(Y) = \frac{2}{3}E(X_1) \end{cases}$	$+\frac{1}{3}\mathrm{E}(X_{1})$	$= \frac{2}{3} \mathrm{E}(X)$	$+\frac{1}{3}\mathrm{E}(X)$	$= \mathrm{E}(X)$		
	= 6(0.35)) + 9(0.65)	0; = 7.95 or	159 20			M1; A1 cao
							[2]

		Question 5 Notes
5. (a)	Note	You can mark parts (a) and (b) together for this question.
	1st B1	At least three correct values for y of either 6, 7, 8 or 9
	2 nd B1	Correct values for y of 6, 7 8 and 9 only. Note: Any extra value(s) given is 2 nd B0.
(b)	1st M1	At least one of either $(0.35)^2$, $(0.65)(0.35)$, $(0.35)(0.65)$ or $(0.65)^2$. Can be implied.
	2 nd M1	At least two of either $(0.35)^2$, $(0.65)(0.35)$, $(0.35)(0.65)$ or $(0.65)^2$. Can be implied.
	1st A1	At least two correct probabilities given which either must be linked
		to a correct sample (x_1, x_2) or their followed through y-value.
	2 nd A1	At least 3 correct probabilities corresponding to the correct value of y.
	B1ft	Either
		• all 4 correct probabilities corresponding to the correct value of y
		• 6, 7, 8 and 9 with two correct probabilities, two other probabilities
		and $\sum p(y) = 1$
	Note	B1ft is dependent on 1 st M1 2 nd M1 1 st A1.
	Note	A table is not required but y-values must be linked with their probabilities for 2 nd A1 B1
	Note	Eg: (6, 6) by itself does not count as an acceptable value of y
(c)	M1	A correct follow through expression for $E(Y)$ using their distribution
. ,	Note	Also allow M1 for a correct expression for $E(X)$
	A1	7.95 cao Allow $\frac{159}{20}$

Question Number	Scheme		Marks	
6. (a)	$X \sim B(30, 0.4)$ $X \sim B(30, 0.4)$			
(b)	Eg: Any one of either	Any one of these two assumptions in context which refers to insurance.	[1 <u>]</u> B1	
<i>(</i>)	P(X < r) < 0.05		[1]	
(c)	$\begin{bmatrix} D(V < 9) - D(V < 0) \end{bmatrix} = 0.0040$	t one of either 0.094(0) or 0.0435 seen in part (c)	M1	
	So $r = 8$	r = 8	A1	
(d)	${Y \sim B(100, 0.4) \approx} Y \sim N(40, 24)$	Normal or N (40, 24)	[2] M1 A1	
	$\left\{ P(Y \ge t) \right\} \approx P(Y > t - 0.5)$ Fo	r either $t - 0.5$ or $t + 0.5$	M1	
	$\left\{ = P\left(Z > \frac{(t - 0.5) - 40}{\sqrt{24}}\right) = 0.938 \right\}$			
	(t-0.5)-40 star	with their mean and their ndard deviation and either 5 or t or $t+0.5$ or $t-1.5$	M1	
	V 24	r awrt –1.54 or awrt 1.54	B1	
	So, $\{So, t = 32.955571\} \Rightarrow \underline{t = 33}$			
(e)	$H_0: p = 0.4, H_1: p < 0.4$ Both hypo	theses are stated correctly	[6] B1	
	$\left\{ \text{Under H}_{0}, X \sim B(25, 0.4) \right\}$			
	Probability Method	$P(X \leqslant 6)$		
	$P(X \le 6) := 0.0736$ $\{P(X \le 7) = 0.1536\}$	Either 0.0736 or $CR: X \leq 6$ or $CR: X < 7$	M1 A1	
	$\{0.0736 < 0.10\}$			
	Reject H ₀ or significant or 6 lies in the CR Dependent on 1 st M1 See notes			
	So <u>percentage</u> (or <u>proportion</u>) who buy <u>insurance</u> has <u>decreased</u> .			
			[5] 15	

Question Number		Scheme	Marks			
6. (e)	Alternati	ve Method: Normal approximation to the Binomial Distribution				
	• N	formal Approximation gives 0.0764 (or 0.07652) and loses all A marks				
	$H_0: p = 0$	0.4, $H_1: p < 0.4$ Both hypotheses are stated correctly	B1			
	${Y \sim B(2)}$	$(5, 0.4) \approx Y \sim N(10, 6)$				
	$P(X \leq 6)$	$P(X \le 6.5)$ $P(X \le 6)$ or $P(X \le 6.5)$	M1			
		R(z = 6.5-10)				
		$= P\left(Z < \frac{6.5 - 10}{\sqrt{6}}\right)$				
		= P(Z < -1.4288)				
		$\{=1-0.9236\} = 0.0764$ Award A0 here	A0			
	{(0.0764 < 0.10				
	Reject	H_0 or significant As in the main scheme	M1			
		ntage (or proportion) who buy insurance has decreased. Award A0 here	$A\theta$			
	-	Question 6 Notes	l .			
6. (a)	B1	$X \sim B(30, 0.4)$ or $X \sim Bin(30, 0.4)$. Condone $X \sim b(30, 0.4)$				
	Note	$X \sim B(30, 0.4)$ o.e. must be seen in part (a) only.				
(b)	B1	For any one of the two acceptable assumptions listed anywhere in part				
()	Note	A contextual statement, which refers to insurance, is required for this m	nark.			
(c)	Note 1st M1	Award M1 A1 for $r = 8$ seen from no incorrect working. For writing N or for using a normal approximation.				
(d)	1 st A1	For a correct mean of 40 and a correct variance of 24				
	Note	1 st M1 and/or 1 st A1 may be implied in applying the standardisation formula				
	2 nd M1	For either $t-0.5$ or $t+0.5$ (i.e. an attempt at a continuity correction)				
	3 rd M1	As described on the mark scheme.				
	B1	-1.54 or 1.54 or awrt -1.54 or awrt 1.54 . Note: Calculator gives -1.5382				
	2 nd A1	t = 33 cao (The integer value is required).				
(e)	B 1	$H_0: p = 0.4, H_1: p < 0.4$ corectly labelled. Also allow $H_0: \pi = 0.4, H_1: \pi < 0.4$				
		Also allow $H_0: \pi = 0.4$, $H_1: \pi < 0.4$ or $H_0: p(x) = 0.4$, $H_1: p(x) < 0.4$				
	Note	B0 for $H_0 = 0.4$, $H_1 < 0.4$				
	1 st M1	Probability Method & CR Method: Stating $P(X \le 6)$				
	1 st A1	Either 0.0736 or CR : $X \le 6$ or CR : $X < 7$ Note: Condone CR ≤ 6				
	Note	Award 1 st M1 1 st A1 for writing down CR : $X \le 6$ or CR : $X < 7$ from no working.				
	Note	Give A0 for stating CR : $P(X \le 6)$				
	2 nd dM1	dependent on the FIRST method mark being awarded.				
		For a correct follow through comparison based on their probability or CR and	their			
		significance level compatible with their <i>stated</i> alternative hypothesis.	annet II "			
	N T .	Do not allow non-contextual conflicting statements. Eg. "significant" and "a	ccept H ₀ "			
	Note	M1 can be implied by a correct contextual statement.				
	2 nd A1	Award for a correct solution only with all previous marks in part (e) being second correct conclusion which is in context, using the words percentage (or proportion).				
		insurance and decreased (or equivalent words for decreased).	<u>.uon</u>),			

Question Number	Scheme	
7. (a)	$\int_0^k \left(\frac{2x}{15}\right) \left\{ dx \right\} + \int_5^k \frac{1}{5} (5-x) \left\{ dx \right\} = 1$ Complete method of writing a correct equation for the area <i>with correct limits</i> and setting the result equal to 1	M1
		M1
	$\left[\frac{x^2}{15}\right]_{\{0\}}^{\{k\}} + \left[x - \frac{x^2}{10}\right]_{\{k\}}^{\{5\}} = 1$ $\mathbf{Both} \ \frac{2x}{15} \to \frac{x^2}{15} \ \mathbf{and} \ \frac{1}{5}(5-x) \to x - \frac{x^2}{10}$	A1 o.e.
	$\left[\left(\frac{k^2}{15} \right) + \left(5 - \frac{5^2}{10} - \left(k - \frac{k^2}{10} \right) \right) = 1$	
	$2k^2 + 150 - 75 - 30k + 3k^2 = 30$	
	$k^2 - 6k + 9 = 0$ or $\frac{k^2}{6} - k + \frac{3}{2} = 0$	
	Dependent on the 1st M mark	
	$(k-3)(k-3) = 0 \Rightarrow k =$ Attempt to solve a 3 term quadratic equation leading to $k =$	dM1
	k=3 $k=3$	A1
		[5]
(b)	$\{\text{mode} = \}$ 3 or states their k value from part (a)	B1 ft [1]
(c)	$\left\{ P\left(X \leqslant \frac{k}{2} \middle X \leqslant k\right) = \frac{P\left(X \leqslant \frac{k}{2} \cap X \leqslant k\right)}{P\left(X \leqslant k\right)} \right\}$	
	$= \frac{P\left(X \leqslant \frac{k}{2}\right)}{P\left(X \leqslant k\right)}$ Either $\frac{P\left(X \leqslant \frac{k}{2}\right)}{P\left(X \leqslant k\right)}$ or $\frac{F\left(\frac{k}{2}\right)}{F(k)}$ seen or implied.	M1
	$= \frac{\int_0^{\frac{k}{2}} \left(\frac{2x}{15}\right) \left\{ dx \right\}}{\int_0^{k} \left(\frac{2x}{15}\right) \left\{ dx \right\}}$ see notes	dM1
	$= \frac{\frac{1}{15} \left(\frac{k}{2}\right)^2}{\frac{k^2}{15}}$ Correct substitution of their limits or their k into conditional probability formula.	A1ft
	$ \left\{ = \frac{\left(\frac{9}{60}\right)}{\left(\frac{9}{15}\right)} = \frac{0.15}{0.6} \right\} = \frac{1}{4} \qquad \frac{1}{4} \text{ or } 0.25 $	A1 cao
		[4]
		10

	Question 7 Notes				
7. (a)	1 st M1	$\int_0^k \left(\frac{2x}{15}\right) \left\{ dx \right\} + \int_5^k \frac{1}{5} (5-x) \left\{ dx \right\} = 1. (with \ correct \ limits \ and = 1) \left\{ dx \right\} \text{not needed.}$			
	2 nd M1	Evidence of $x^n \to x^{n+1}$			
	1 st A1	Both $\frac{2x}{15} \rightarrow \frac{x^2}{15}$ and $\frac{1}{5}(5-x) \rightarrow x - \frac{x^2}{10}$			
	3rd dM1	dependent on the FIRST method mark being awarded.			
		Attempt to solve a three term quadratic equation. Please see table on page 20			
	2 nd A1	k = 3 from correct working.			
	Note WARNING: $\frac{2x}{15} = \frac{1}{5}(5-x)$ to get $k = 3$ is M0M0A0M0A0. Note It is possible to give M0M1A1M0A0 in part (a).				
(b)	B1 ft	• • • • • • • • • • • • • • • • • • • •			
(c)	1 st M1	Either $\frac{P\left(X \leqslant \frac{k}{2}\right)}{P\left(X \leqslant k\right)}$ or $\frac{F\left(\frac{k}{2}\right)}{F(k)}$, seen or implied by their later working.			
	Note	Without reference to a correct conditional probability statement give 1 st M0 for either			
	$\frac{f\left(\frac{k}{2}\right)}{f(k)} \text{ or } \frac{F(k) - F\left(\frac{k}{2}\right)}{F(k)} \text{ or } \frac{P\left(X \leqslant \frac{k}{2}\right) \times P\left(X \leqslant k\right)}{P\left(X \leqslant k\right)}$				
	2 nd dM1	1			
		Applies the conditional probability statement by writing down			
	Applies the conditional probability statement by writing down $ \frac{\int_{0}^{\frac{k}{2}} \left(\frac{2x}{15}\right) \left\{ dx \right\}}{\int_{0}^{k} \left(\frac{2x}{15}\right) \left\{ dx \right\}} \text{ with limits.} $ • $ \frac{F\left(\frac{k}{2}\right)}{F(k)} \text{ where } F(x) \text{ is defined as } F(x) = \frac{x^{2}}{15} $ These statements can be implied by later working.				
	Note Finding $P(X \le 1.5) = 0.15$ and $P(X \le 3) = 0.6$ without applying $\frac{0.15}{0.6}$ is 2^{nd} M0				
	1 st A1ft Note	1ft Correct substitution of their limits or their <i>k</i> into conditional probability formula.			
	2 nd A1	$\frac{1}{4}$ or 0.25 cao			
	Note	Condone giving 2 nd A1 for achieving a correct answer of 0.25 where at least one of their			
		stated $P\left(X \leqslant \frac{k}{2}\right)$ or $P\left(X \leqslant k\right)$ is greater than 1			
	Note				
		This can score 4 marks.			

7. (a)	Alternative Method 1 for Part (a) Using the CDF				
	$0 \leqslant x \leqslant k, \ F(x) = \int_0^k \frac{2t}{15} \left\{ dt \right\} = \left[\frac{2t^2}{\underline{30}} \right]_0^x = \frac{x^2}{\underline{15}}$ Evidence of $x^n \to x^{n+1}$	2 nd M1			
	$k < x \le 5$, $F(x) = F(k) + \int_{k}^{x} \frac{1}{5} (5-t) \{dt\}$ Both $\frac{2x}{15} \to \frac{x^{2}}{15}$ and	1 st A1			
	$= \frac{k^2}{15} + \left[\frac{1}{5} \left(5t - \frac{t^2}{2} \right) \right]_k^k \qquad \qquad \frac{1}{5} (5 - x) \to \underline{x - \frac{x^2}{10}}$	o.e.			
	$= \frac{k^2}{15} + \frac{1}{5} \left(\frac{5x - \frac{x^2}{2}}{2} \right) - \frac{1}{5} \left(5k - \frac{k^2}{2} \right)$				
	$= x - \frac{x^2}{10} - k + \frac{k^2}{6}$ Complete method of writing a correct				
	$\left\{ F(5) = 1 \Rightarrow \right\} 5 - \frac{5^2}{10} - k + \frac{k^2}{6} = 1$ equation for the area with correct limits and setting $F(5) = 1$	1 st M1			
F ()	then apply the main scheme				
7. (a)	Alternative Method 2 for Part (a) Use of Area	N 1			
	$\frac{1}{2}k\left(\frac{2k}{15}\right) + \frac{1}{2}\left(\frac{5-k}{5}\right)(5-k) = 1$ Complete area expression put = 1 At least one term correct on LHS Correct LHS	M1 M1 A1 o.e.			
	then apply the main scheme				
General	Note The c.d.f is defined as				
	$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{15}, & 0 \le x \le 3 \\ x - \frac{x^2}{10} - \frac{3}{2}, & 3 < x \le 5 \\ 1, & x > 5 \end{cases}$				
7. (a)	Method mark for solving a 3 term quadratic of the form $x^2 + bx + c = 0$				
	Factorising/Solving a quadratic equation is tested in Question 7(a).				
	1. Factorisation				
	$(x^2 + bx + c) = (x + p)(x + q)$, where $ pq = c $, leading to $x =$				
	$(a x^2 + bx + c) = (m x \pm p)(n x \pm q)$, where $ pq = c $ and $ mn = a $, leading to $x =$ 2. Formula Attempt to use correct formula (with values for a , b and c)				
	3. Completing the square				
	Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x =$				

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